Digital SAR Processing Using a Fast Polynomial Transform

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In this paper, a new digital processing algorithm based on the fast polynomial transform is developed for producing images from Synthetic Aperture Radar data. This algorithm enables the computation of the two-dimensional cyclic correlation of the raw echo data with the impulse response of a point target, thereby reducing distortions inherent in one-dimensional transforms. This SAR processing technique was evaluated on a general-purpose computer and an actual Seasat SAR image was produced. However, regular production runs will require a dedicated facility. It is expected that such a new SAR processing algorithm could provide the basis for a real-time SAR correlator implementation in the Deep Space Network.

I. Introduction

The Venus Orbiting Imaging Radar (VOIR) mission planned for the late 1980's will require handling high rate ($>$ 2–4 msps) telemetry containing synthetic aperture radar (SAR) data. In the planning for upgrading the current DSN system to handle the increased data rates, consideration must be given to how SAR data will affect telemetry validation and monitoring functions. Generally, signal-to-noise and error rate estimates provide sufficient inputs for telemetry monitoring. However, these estimates by themselves are not always good indicators of image quality; direct monitoring of images is a more effective method of detecting telemetry degradation. For missions in which images are readily reconstructed from telemetry data such as in optical imaging, this kind of monitoring is accomplished by real-time image processing at JPL, which takes only a few minutes' delay. However, even in the "quick-look" mode for VOIR SAR data processing, the delay between telemetry reception and image production will be greater than three hours – an unacceptable delay considering that a station can receive for only about eight hours a day. A better method to reduce the time between SAR data reception and image production to an acceptable value must be developed in order to validate the Deep Space Network (DSN) performance. One technique would be to locate a low-cost quick-look SAR processor at each complex or one at the Network Operations and Control Center (NOCC).

Section II explains the background and the concept of a new technique for digital SAR processing. In Section III, it is explained how the range offset data were conveniently translated to baseband for processing. In Section IV, the formulation of the two-dimensional impulse response of a point target is developed. In Section V, some of the data management considerations for using the fast polynomial transform (FPT) for two-dimensional correlation are treated. Finally, Section VI explains how multiple looks are registered and combined and presents results of SEASAT SAR imagery. Section VII presents the conclusions of this report.

II. Background

Earlier work (Ref. 1) has shown that a VOIR processor handling data from the high-resolution mode must compensate for range migration. When range migration occurs, the two-dimensional impulse response from a point target does not factor into the product of two one-dimensional impulse responses. Currently, SEASAT SAR data accommodates the range migration feature by the digital hybrid correlation algorithm of Wu (Ref. 2). In brief, the azimuth SAR filter is approximated by several linear segments. Then the SAR data corresponding to each segment is correlated separately. Finally the results are coherently summed to produce the full resolution image. The advantage of Wu's hybrid algorithm is that the Fourier transform of the azimuth matched filter is one-dimensional. Hence, one-dimensional FFT can be used to compute azimuth correlation. However, when the two-dimensional impulse response does not change too rapidly with range, full two-dimensional correlation of SAR data with the proper impulse response would eliminate the need for the approximating, linear segments of the hybrid algorithm. Nussbaumer, Quandalle, Arambepola, and Rayner (Refs. 3, 4), and more recently, Truong, Reed, Lipes, and Wu (Ref. 5) showed that a radix-2 FPT could be used to efficiently compute a two-dimensional correlation. It was shown (Ref. 6) that a combination of the FPT and the CRT could be used to very efficiently compute a two-dimensional cycle convolution of $d_1 \times d_2$ array of complex numbers, where $d_2 = 2^m$ and $d_1 = 2^{m-r+1}$ for a $1 \leqslant r \leqslant m$. This FPT convolutional algorithm requires considerably fewer multiplications and about the same number of additions that the usual FFT method needs for the two-dimensional case. It is shown (Ref. 6) that this new algorithm can be implemented readily on a digital computer. It is demonstrated also that the speed of this algorithm is approximately 25% faster than the conventional FFT method for computing two-dimensional convolutions.

To investigate the FPT approach to correlate SAR data with significant range migration, SEASAT SAR data having the characteristics listed in Appendix A was processed on a general-purpose computer. An approach capable of compensating for the magnitude of range migration of the SEASAT data can easily handle VOIR high-resolution data range migration. The main purpose of this effort was to uncover features of the FPT approach that might prevent a real-time implementation and to determine qualitatively whether the range-dependent variation in impulse response would affect image quality. The important real-time processing considerations of autofocusing and Doppler centroid determination have not been treated and will be deferred to later work. The values of Doppler center frequency and Doppler rate used for the impulse response have been supplied to us by C. Wu. A flow diagram of the digital processing system is shown in Fig. 1. Raw SEASAT, range offset data representing 5120 azimuth cells by 3072 range cells were input.

III. Translation of Offset Spectrum of the Raw Range Data to Baseband

In general, the basic geometry of SEASAT SAR is shown in Fig. 2. The antenna of the uncertainties in the SAR attitude flies at a height h above the moving Earth's surface with the center of beam (boresight) making an incident angle; i.e., $\beta = 20^\circ$ with nadir. The direction along which the spacecraft moves is called the azimuth or x direction and the distance in the direction to a point scatterer as measured from the antenna is called the range or r direction.

The SEASAT SAR transmits a pulse waveform of the form $S(t) = a(t) \cos(2\pi f_0 t + \pi bt^2)$ where $a(t)$ is a rectangular pulse waveform with pulse length τ, f_0 is the frequency of the coherent carrier and πbt^2 is the term needed for linear frequency modulation. Let T_1 be the pulse repetition time. In Fig. 2, one observes that if the SEASAT SAR transmits the nth pulse, the distance from the SEASAT SAR to a point scatterer location at $(t_0, r(0))$ is $r(nT_1)$, where $r(0)$ is the minimum distance from a point scatterer to the antenna. The returned signal from the nth pulse is of form

$$S\left(t - nT_1 - 2\frac{r(nT_1)}{c}\right) = \sigma G\left(\frac{x_0 - nT_1 v}{r(0)}\right) a\left(t - nT_1 - \frac{2r(nT_1)}{c}\right) \cos\left[2\pi f_0 \left(t - nT_1 - \frac{2r(nT_1)}{c}\right) + \pi b \left(t - nT_1 - \frac{2r(nT_1)}{c}\right)^2\right] \quad (1)$$

where σ is the radar cross section of a point scatterer location at $(x_0, r(0))$, f_0 is the transmitter frequency, c is the speed of light, $G(\theta)$ is the physical antenna pattern in azimuth direction θ, and the pulse is

$$a\left(t - nT_1 - \frac{2r(nT_1)}{c}\right) = \begin{cases} 1 & \text{if } 0 \leqslant t - nT_1 - \dfrac{2r(nT_1)}{c} \leqslant \tau \\ 0 & \text{otherwise} \end{cases}$$

Let $t_1 = nT_1$ and $t_2 = t - nT_1$ for that integer n such that $0 \leqslant t_2 \leqslant T_1$. Then (1) becomes

$$S\left(t_2 - \frac{2r(t_1)}{c}\right) = \sigma G\left(\frac{x_0 - vt_1}{r(0)}\right) a\left(t_2 - \frac{2r(t_1)}{c}\right)$$

$$\cos\left[2\pi f_0\left(t_2 - \frac{2r(t_1)}{c}\right) + \pi b\left(t_2 - \frac{2r(t_1)}{c}\right)^2\right] \quad (2)$$

where $r(t_1) = r(nT_1)$ and

$$a\left(t_2 - \frac{2r(t_1)}{c}\right) = \begin{cases} 1\,, & \text{if } 0 \leqslant t_2 - \frac{2r(t_1)}{c} \leqslant \tau \\ 0\,, & \text{otherwise} \end{cases}$$

In SEASAT-A SAR, the range data constitutes a block of 4096 samples of raw data, each of 4-bits. In order to convert this real sampled data, the range data, into complex sampled data, advantage is taken of the fact that spectrum is offset from baseband by frequency $f_s/4$, where $f_s = 45.53$ MHz is the sampling rate of the real raw radar data. A method of translating the spectrum to baseband is now described.

If the radar footprint is regarded as a collection of distributed point scatterers, the returned signal has the form

$$x(t_2) = \sum_j \sigma G\left(\frac{x_j - vt_1}{r_j(0)}\right) a\left(t_2 - \frac{2r_j(t_1)}{c}\right) \cos\left[2\pi \frac{f_s}{4}\left(t_2 - \frac{2r_j(t_1)}{c}\right) + \pi b\left(t_2 - \frac{2r_j(t_1)}{c}\right)^2\right] \quad (3)$$

where $G(\theta)$ is the gain of the antenna and $r_j(0)$ is the minimum of the distance from the jth point scatterer to the antenna, and $r_j(t_1)$ is the distance from the antenna to the jth point scatterer within the footprint. Sampling $x(t_2)$ in (3) at intervals $T_2 = 1/f_s$, yields

$$x(kT_2) = \sum_j \sigma G\left(\frac{x_j - vt_1}{r_j(0)}\right) a\left(kT_2 - \frac{2r_j(t_1)}{c}\right) \cos\left[2\pi \frac{f_s}{4}\left(kT_2 - \frac{2r_j(t_1)}{c}\right) + \phi_j(kT_2)\right], \quad 0 \leqslant k \leqslant 4095 \quad (4)$$

where

$$\phi_j(kT_2) = \pi b\left(kT_2 - \frac{2r_j(t_1)}{c}\right)$$

is the sampled data sequence. Since $T_2 = 1/f_s$ for even and odd k, this formula for $x(kT_2)$ in (4) can be expressed as follows:

$$x(2nT_2) = (-1)^n \sum_j \sigma G\left(\frac{x_j - vt_1}{r(0)}\right) a\left(2nT_2 - \frac{2r_j(t_1)}{c}\right) \cos\left[\frac{2\pi f_s}{4}\frac{2n}{f_s} + \phi_j(2nT_2)\right]$$

$$= (-1)^n \sum_j \sigma_j G\left(\frac{x_j - vt_1}{r_j(0)}\right) a\left(2nT_2 - \frac{2r_j(t_1)}{c}\right) \cos \phi_j(2nT_2) \quad \text{for } n = 0, 1, 2, \ldots\ldots. \quad (5a)$$

Similarly,

$$x[(2n+1)T_2] = (-1)^{2n+1} \sum_j \sigma G\left(\frac{x_j - vt_1}{r_j(0)}\right) a\left((2n+1)T_2 - \frac{2r_j(t_1)}{c}\right) \sin \phi_j\,[(2n+1)T_2] \quad \text{for } n = 0, 1, 2, \ldots \quad (5b)$$

In (5a), one observes that $x(2nT_2)$ is a cosine sequence for $n = 0, 1, 2, \ldots$. In order to convert $x(2nT_2)$ given in (5a) into complex sequence $Z(2nT_2)$, the imaginary part of $Z(2nT_2)$, i.e., a sine sequence, is required to compute for summing to cosine sequence given in (5a). This sequence denoted by $x'(2nT_2)$ is obtained by interpolating the samples of sine sequence $x[(2n+1)T_2]$ given in (5b) for $n = 0, 1, 2, \ldots$. Evidently, from the sum of these sine and cose sequences, the appropriate complex data stream is obtained by forming

$$Z(kT_2) = -[x(2kT_2) + jx'(2kT_2)]$$

$$= \sum_j \sigma G\left(\frac{x_j - vt_1}{r_j(0)}\right) a(2kT_2)e^{-i\phi_j(2kT_2)}$$

$$\text{for } 0 \leqslant k \leqslant 1023 \tag{6}$$

Thus, each range sweep of data has been converted to a stream of 2048 complex data points.

IV. The Design of the Two-Dimensional SAR Impulse Response of a Point Target

The returned signal of a point scatterer located at (x_0, $r(0)$) is given in Eq. (2). If the moving antenna radiates the successive pulses to a point scatterer on Earth location at (x_0, $r(0)$), then t_1 in $r(t_1)$ given in (2) lies between $-T/2$ and $T/2$, where T is the total amount of time a point scatterer is in the mainbeam. Consider the effects of a point scatterer on moving Earth and the effects of the uncertainties in the SAR attitude. Then, using the Taylor series expansion, $r(t_1)$ in (2) about $t_1 = 0$ is given by

$$r(t_1) = r(0) + \dot{r}(0)t_1 + \frac{\ddot{r}(0)}{2}t_1^2 \quad , \quad |t_1| \leqslant T/2 \tag{7}$$

$r(t_1)$ in (7) can be rewritten as

$$r(t_1) - r(0) = \frac{\lambda}{2}\left[f_d t_1 + \frac{1}{2}\dot{f}_d t_1^2\right] \quad , \quad |t_1| \leqslant T/2 \tag{8}$$

where $f_d = 2\dot{r}(0)/\lambda$ and $\dot{f}_0 = 2\ddot{r}(0)/\lambda$ are called the doppler center frequency and the doppler center frequency rate, respectively.

In SEASAT SAR, the values of f_d and $\dot{f}_d$ for the Los Angeles/Burbank area are -1415 Hz and -515.1 Hz/sec, respectively. Using the parameters given in Appendix A, one obtains the azimuth integration time to be $T = 25$ sec and $\lambda = 0.235$ (see Ref. 1). Thus, (8) becomes

$$r(t_1) - r(0) = 0.1175\ [-1425\, t_1 - 257.55\, t_1^2], |t_1| \leqslant 1.25 \text{ sec} \tag{9}$$

Assume that the returned signals from the successive pulses are recorded side-by-side. The result forms a two-dimensional impulse response of a point scatterer location at (x_0, $r(0)$) with range $r = c\cdot t_2/2$ along one axis and azimuth $x = vt_1$ along the other axis.

To define a 2-D point scatterer region of a matched filter, i.e., R, consider a fixed azimuth location $t_1 = nT_1$; then the range time corresponding to t_1 is $t_2 = 2r(t_1)/c$. From (2), the returned signal from the nth pulse, $t_1 = nT_1$, only lies in the range time between $t_2 = 2r(t_1)/c$ and $t_2 = \tau + 2r(t_1)/c$. Assume that the moving antenna with uncertainties in the SAR attitude radiates the successive pulses to a point scatterer on moving Earth. From (9) one observes that the azimuth time is located between $|t_1| \leqslant T/2 = 1.25$ sec, where $T = 25$ sec is the total amount of time a point scatterer is in the mainbeam. Thus, R is determined by the pulse width and the pattern of the antenna beam. A 2-D impulse response of a point scatterer only lies in the location between $|t_1| \leqslant 1.25$ sec and $2r(t_1)/c \leqslant t_2 \leqslant 2r(t_1)/c + \tau$. In fact, a two-dimensional matched filter excluding the effect of the physical antenna is

$$h(t_1,t_2) = S\left(t_2 - \frac{2r(t_1)}{2}\right)$$

$$= \begin{cases} e^{-i4\pi r(t_1)/\lambda} \cdot e^{i\pi b(t_2 - 2r(t_1)/c)^2}, & (t_1,t_2)\,\epsilon R \\ 0 & \text{, otherwise} \end{cases} \tag{10a}$$

where R is the 2-D point scatterer region, $\exp\{-i4\pi r(t_1)/\lambda\}$ and $\exp\{i\pi b(t_2 - 2r(t_1)/c)^2\}$ are the azimuth and range matched filters, respectively, and $\lambda = f_0/c$, t_1 and t_2 are in azimuth and range axis, respectively.

Consider the effect of range migration for a matched filter in SEASAT SAR. For the case $t_1 = 1.25$ sec, (9) becomes

$$r(-1.25) - r(0) = 0.1175\ [1425\,(-1.25) - 257.55\,(-1.25)^2]$$

$$= -162.0 \text{ m}$$

In a similar fashion, $r(1.25) - r(0) = 256.25$ m. Hence the range migration is

$$\Delta r = r(1.25) - r(-1.25) = 256.25 - (-162.0) = 418.25 \text{ m}$$

Since the range migration is greater than the range resolution given in Appendix A, i.e., $\Delta r \gg 6.6$ m, the impulse response of a point target at location ($x_0 = 0$, $r(0)$) must be two-dimensional. With the parameters given in Appendix A, the size of this two-dimensional matched filter can be computed to be 4096×832 (see Ref. 1). In SEASAT SAR, $b = 0.562 \times 10^{12}$ cycles/sec^2. Thus, substituting b and

$r(t_1) - r(0)$ given in (9) into (10a), a two-dimensional matched filter is of form

$$h(t_1,t_2) = \begin{cases} e^{i2\pi(f_d t_1 + \dot{f}_d t_1^2/2)} \cdot e^{ib\pi[t_2 - a(f_d + \dot{f}_d t_1^2/2)]^2}, & (t_1,t_2)\,\epsilon R \\ 0 & \text{otherwise} \end{cases} \quad (10b)$$

where f_d = 1425 Hz, $\dot{f}_d$ = 515.1 Hz/sec, $b = 0.562 \times 10^{12}$ cycles/sec^2, a = 0.079 sec, and the 2-D filter region R is located between $|t_1| \leq 1.25$ sec and $2r(t_1)/c \leq t_2 \leq 2r(t_1)/c + 33.8$ μsec and is shown in Fig. 3. Since the number of looks for SEASAT SAR is four, the size of the filter per look is a 1024 X 832 array of complex samples (see Fig. 3).

V. Fast Polynomial Transform for Computing a Two-Dimensional Correlation

In the previous section, one observed that the two-dimensional filter per look is a 1024 X 832 array of complex samples. If the 1024 X 832 filter is extended with zeroes to the size of a 2048 X 2048 array of complex data, then it can be correlated with a 2048 X 2048 array of raw data to obtain a 256 X 1216 image for each look with 6.6- X 25-m resolution in range and azimuth. This two-dimensional cyclic correlation of a 2048 X 2048 array of complex data can be computed by a radix-2 FPT. The radix-2 FPT algorithm is given in Refs. 5 and 6.

VI. Registration and the Combining of Looks

In order to produce an image, one needs to correlate the raw data with the matched filter given in Eq. (10b). This procedure can be accomplished by using a 4-look overlap-save FPT procedure. The procedure currently being used for this 4-look FPT procedure is the following.

A 6144 X 2048 array of complex raw data is taken. It is divided into 1024 X 2048 pieces denoted by A, B, C, D, E, F. The matched filter is a 4096 X 832 array of complex data. It is divided into 1024 X 832 pieces denoted by looks 1, 2, 3, 4 (see Fig. 3). Each look is extended to a 2048 X 2048 with zeroes. The desired effect is to correlate the complete raw data with the complete matched filter, producing a 2048 X 1216 picture. Since we only have the capability of performing a 2048 X 2048 cyclic correlation, AB is correlated with look 1 to obtain a 1024 X 1216 array of real imagery data that has an azimuth resolution of 25 m for one look. Similarly, look 2 is correlated with BC, look 3 is correlated with CD, and look 4 is correlated with DE. Then these resulting looks are summed to obtain the first 1024 X 1216 image for all looks with 25-m azimuth resolution. To obtain another 1024 X 1216 image, looks 1, 2, 3, and 4 are correlated with BC, CD, DE, and EF, respectively. These resulting looks are summed to obtain the second 1024 X 1216 image for all looks with 25-m azimuth resolution. Hence, a 2048 X 1216 array of real imagery data with 6.6- X 25-m range and azimuth resolution is obtained. Since a 4-look overlap-save FPT is used in this procedure, the actual picture is (2048/4) · 1216 = 512 X 1216 array of real data with 6.6- X 25-m resolution image for all looks.

Using the same procedure described above, a 4-look SEASAT SAR 512 X 1216 array of real imagery data of the Los Angeles/Burbank area with 6.6- X 25-m resolution is shown in Fig. 4.

VII. Conclusions

An FPT has been used for digital SAR processing and an SAR image has been generated. Moreover, the FPT was shown to have an architecture suitable for hardware implementation. These advantages make this FPT algorithm a good candidate for developing a real-time SAR processor.

Acknowledgment

The authors express thanks to Dr. I. S. Reed for many helpful suggestions on the subject of SAR and to Dr. C. Wu for supplying them with the raw SEASAT data and corresponding reference functions.

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Appendix A
Seasat SAR Sensor Characteristics

SAR System Parameters (SEASAT-A SAR)

SAR orbit	SEASAT-A Orbit
Nominal altitude	794 km
Nominal speed	7450 m/sec
Transmitter frequency	1275 MHz
Pulse repetition frequency	1463, 1537, 1645 Hz
Pulse width	33.8 μsec
Pulse bandwidth	19 MHz
A/D rate for range offset signals	45.53 MHz
A/D window	288 μsec
Antenna dimension	2 m $\times$ 10.5 m
Antenna look angle	20° cone
Attitude (roll, pitch, yaw) accuracy	$\pm 0.5^\circ$
Image dynamic range	50 dB
Image resolution (range and azimuth)	6.6 m/25 m
Number of looks	4

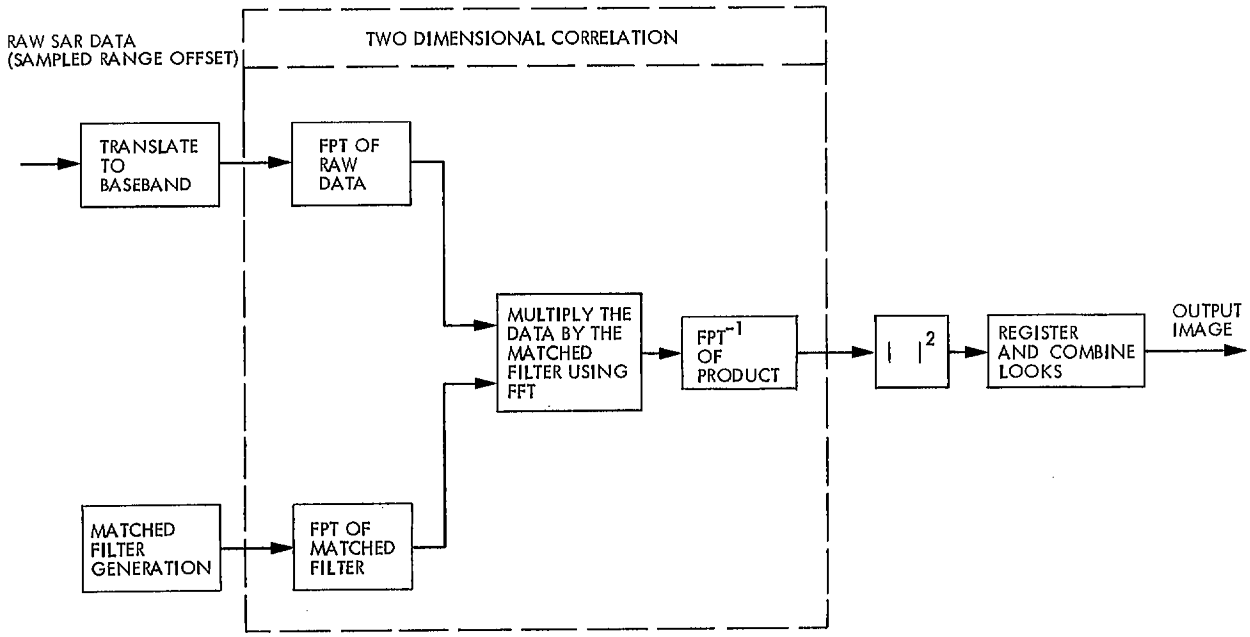

Fig. 1. Flow diagram of digital SAR processing system using the fast polynomial transform

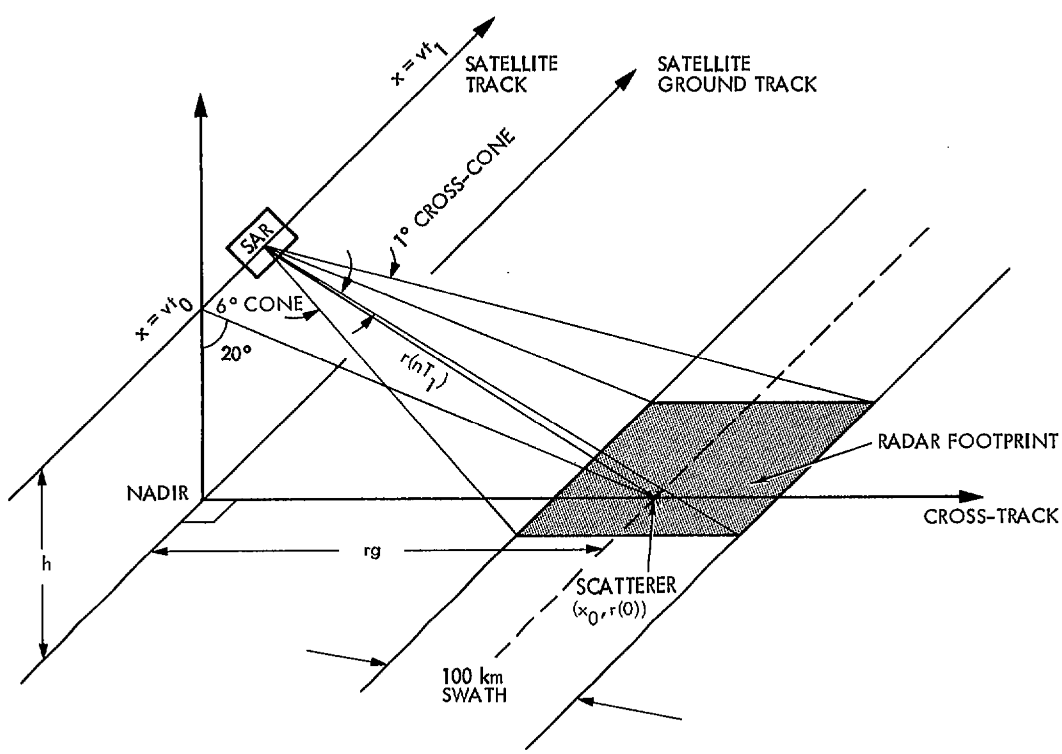

Fig. 2. SEASAT SAR imaging geometry from the *n*th pulse

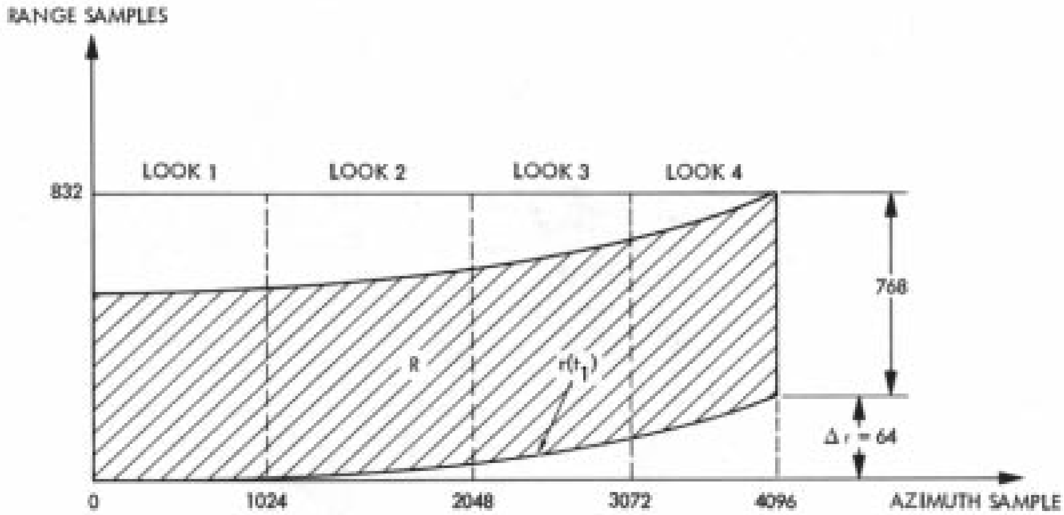

Fig. 3. A range migration and the 2-D region R of a matched filter for SEASAT SAR processing

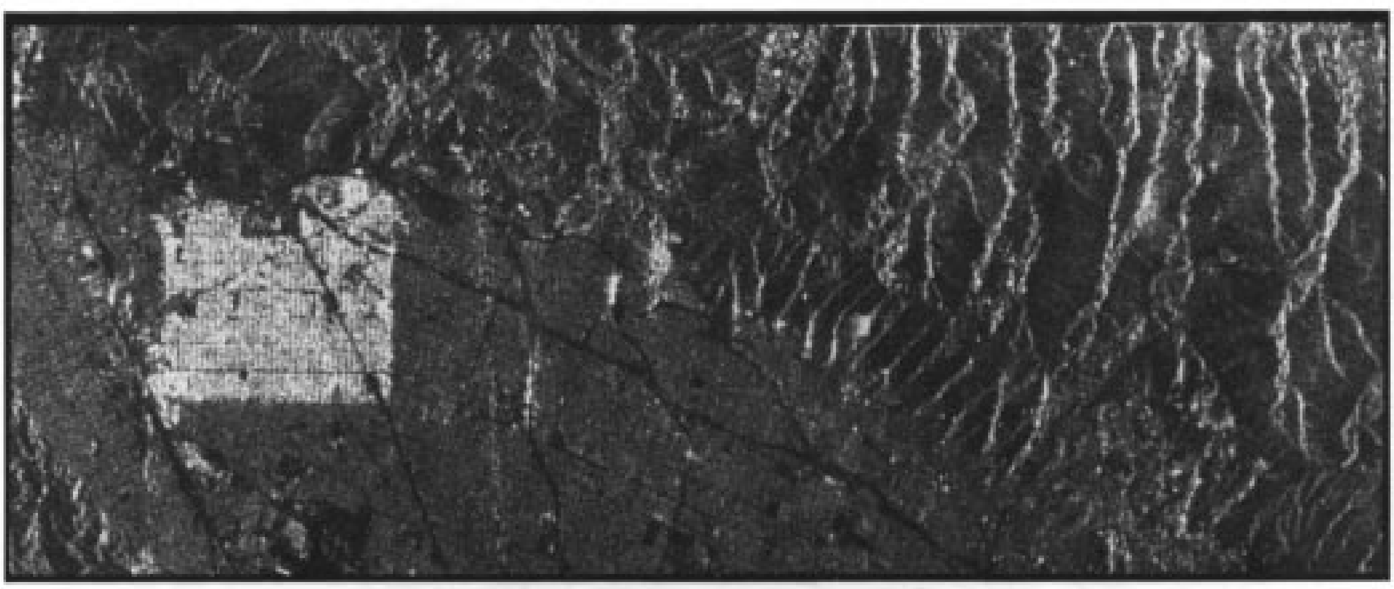

Fig. 4. A 512 × 1216 array of real imagery data of the Los Angeles/Burbank area with 4-look SEASAT SAR 6.6- × 25-m resolution